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PREFACE.

The design of the present work is to supply beginners with a cheap and easy edition of the first Book of Euclid. The object of the Author in preparing these pages, has been to make the demonstrations as strict and concise as possible, and at the same time to reduce them to the most simple form. In order to afford every facility to the Learner, each demonstration is begun and completed on the same page with the figure to which it refers; so that the inconvenience arising from having the demonstration on one page, and the figure to which it refers very often on another, is completely removed.

Prefixed to the work is an historical Introduction, showing the rise and progress of this science, with the explanations of the principal terms; and at the end is a selection of useful Problems and Theorems as Exercises for Pupils.

HISTORICAL INTRODUCTION.

GEOMETRY is a science which teaches and demonstrates the properties of lines, angles, surfaces, and solids. The word is from the Greek, and signifies "to measure the earth," it being the necessity of measuring the earth, or portions of land, that first led to the invention of the principles and rules of this art, which has since been extended and applied to magnitudes of all kinds of extension. It is generally believed that the Egyptians have been the inventors of Geometry, and that the annual inundations of the Nile have occasioned the invention. For as that river overflowed its banks, and covered the whole face of the country, it destroyed all landmarks, and confounded the boundaries of men's estates; so that the people were laid under the necessity of inventing certain methods and measures, to enable them to distinguish and adjust the limits of their respective lands, when the waters were withdrawn.

And so highly valued was this science among the Egyptians, that as soon as youths could read, they were taught Geometry and Arithmetic with the greatest possible care. Plato held this science in such high esteem, that on the door of his Academy was read this inscription, "Let none ignorant of Geometry enter here." Another instance which proves how highly it was valued by men of former ages, is, that when a certain person, who was ignorant of Geometry, was desirous to be Xenocrates' auditor, he said, "Go thy way, for thou wantest the very handle to Philosophy."

The writer who has contributed most to the improvement and refinement of this science, is the celebrated Euclid, professor in the Academy at Alexandria, whose Elements will be admired as long as a taste for true science remains. Euclid flourished about 290 years before the Christian era, and of his Elements we have abundance of editions and comments at the present day.

EXPLANATIONS OF TERMS.

GEOMETRY proceeds by means of Definitions, Postulates, Axioms, and Propositions.

A Definition is the explanation or meaning of a term of art; or describes a magnitude by enumerating its properties.

A Postulate is a petition or demand, necessary to be granted, admitted as possible.

An Axiom is a self-evident truth, which requires no proof to confirm it.

A Proposition is a sentence in which something is affirmed or proposed to be done; it is either a theorem, or a problem.

A Theorem is a proposition which proposes some truth to be proved; it wants demonstration.

A Problem is a proposition which proposes something to be done or constructed; it requires solution.

The Construction is the drawing of certain lines or figures, to enable us to demonstrate the theorem or to solve the problem.

The Demonstration is the reasoning employed to show that the theorem is true, or that the problem is solved.

A Corollary is a consequent truth, deduced from a foregoing demonstration.

A Scholium is a remark made upon a foregoing proposition.

A Lemma is a minor proposition, necessary to be demonstrated, previous to a more important one which follows it.

In a theorem certain things are supposed and admitted, from which a conclusion is to be drawn; this supposition is called the hypothesis.

Some propositions are proved directly by means of definitions, axioms, or propositions already demonstrated.

Other propositions are proved indirectly, by showing that an other supposition or hypothesis than the one advanced woul lead to an absurdity or impossibility.

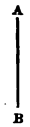
The sentence which expresses the substance of a proposition is called the Enunciation.

DEFINITIONS.

1. A point is that which has no parts.

2. A line is length without breadth or thickness; as, the line A B.

3. The extremities of a line are points; as, the points A and B of the line A B.

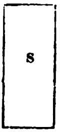


4. A straight or right line is the shortest distance between two points, and lies evenly between them.

5. A superficies or surface is that which has length and breadth, but no thickness; as, S.

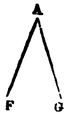
6. The extremities of a superficies are lines.

7. A plane superficies is that which lies evenly between its extreme lines.



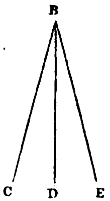
8. A rectilineal angle is the opening or corner between two right lines, which meet in a point, but do not form one right line; as, the angle A.

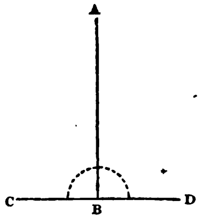
9. The right lines which form an angle, are called the sides; and the point in which the sides me is called the vertex of the angle; as, the vertex A.



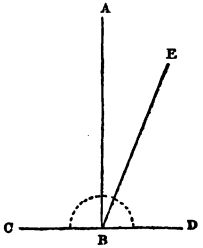
10. When several angles meet in one point, each angle is expressed by three letters, of which the middle letter is placed at the vertex of the angle, and another upon each of the sides which form the angle; thus, the angle formed by the lines C B and D B is called the angle C B D or D B C; the angle formed by the lines C B and E B is called the angle C B E or E B C.

N.B. When there is only one angle at a point, it may be expressed by a single letter; as, the angle A or by three letters as, F A G.





11. When a right line standing upon another right line makes the adjacent angles equal to one another, each of them is called a right angle; and the right line which stands upon the other is called a perpendicular to it; thus, if the line A B, standing upon C D, makes the angles A B C and A B D equal, each of them is a right angle, and A B is perpendicular to C D.

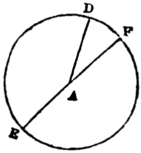


12. One angle is said to be greater than another, when the lines which form the angle are farther from each other than the lines which form the other, measuring at equal distances from the vertex of each; thus, the angle E B C is greater than the angle A B C.

13. An angle greater than a right angle is called obtuse; as, the angle E B C.

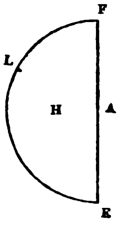
14. An angle less than a right angle is called acute; as, the angle E B D.

15. A plane figure is a space inclosed by one or more lines.



16. A circle is a plane figure bounded by one line called the circumference, and is of such a kind, that all right lines drawn from a certain point within the figure to the circumference are equal to one another; as, the circle D E F.

17. This point is called the centre of the circle, and the right line drawn from the centre to the circumference is called the radius; as, the radius A D drawn from the centre A: if more lines than one be drawn, they are called the radii.



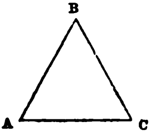
18. A diameter of a circle is a right line passing through the centre, and terminated both ways by the circumference; as, the diameter E F.

19. A semicircle is the figure contained by the diameter and half the circumference; as, H.

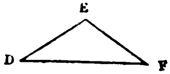
20. An arc is any part of the circumference; as, E L.

21. A figure bounded by right lines is called a rectilineal figure; if it be bounded by three lines it is called a triangle; if bounded by four lines it is called a quadrilateral; if bounded by more than four lines it is called a polygon.

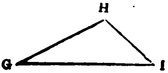
22. Triangles are of three kinds, when described according to the nature of their sides; namely, Equilateral, Isosceles, and Scalene.



23. An equilateral triangle is that which has three equal sides; as, the triangle A B C.

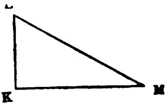


24. An isosceles triangle is that which has two equal sides; as, the triangle D E F.

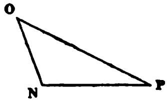


25. A scalene triangle is that which has three unequal sides; as, the triangle G H I.

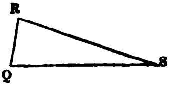
26. Triangles are also of three kinds, when described according to the nature of their angles; namely, Right-angled, Obtuse-angled, and Acute-angled.



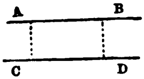
27. A right-angled triangle is that which has one of its angles a right angle; as, the triangle K L M.



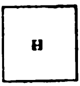
28. An obtuse-angled triangle is that which has one of its angles an obtuse angle; as, the triangle N O P.



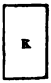
29. An acute-angled triangle is that which has all its angles acute; as, the triangle Q R S.



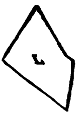
30. Parallel right lines are such as are equidistant from each other, and if produced would never meet; as, A B and C D.



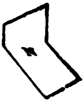
31. A square is a quadrilateral figure, having all its sides equal and all its angles right angles; as, H.



32. An oblong or rectangle is a quadrilateral figure, having all its angles right angles, but its length exceeds its breadth; as, K.



33. A rhombus is a quadrilateral figure having all its sides equal, but its angles are not right angles; as, L.



34. A rhomboid is a quadrilateral figure *having* the opposite sides equal, but its *angles* are not right angles; as, N.

35. All quadrilateral figures, whose opposite sides are parallel, are called parallelograms; and the line joining two of its opposite angles, is called the diagonal or diameter.

Postulates.

1. Let it be granted that a right line may be drawn from any one point to any other point.

2. That a terminated right line may be produced to any length in a right line.

3. That a circle may be described from any centre, with any distance from that centre as radius.

Axioms.

1. Things which are equal to the same, are equal to one another.

2. If equals be added to equals, the wholes are equal.

3. If equals be taken from equals, the remainders are equal.

4. If equals be added to unequals, the wholes are unequal.

5. If equals be taken from unequals, the remainders are unequal.

6. Things which are double of the same, are equal to one another.

7. Things which are halves of the same, are equal to one another.

8. Magnitudes which coincide with one another, that is, which exactly fill the same space, are equal to one another.

9. The whole is greater than its part.

10. Two right lines cannot inclose a space.

11. All right angles are equal to one another.

12. If a right line meet two other right lines, so as to make the two interior angles on the same side of it taken together less than two right angles, these two right lines being continually produced, shall at length meet on that side on which are the angles, which are less than two right angles.

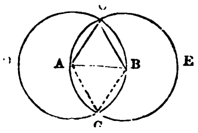
EXPLANATIONS OF CONTRACTIONS.

Ax. for Axiom.
Const. „ Construction.
Cor. „ Corollary.
Def. „ Definition.
Hyp. „ Hypothesis.
Post. „ Postulate.
Prob. „ Problem.
Prop. „ Proposition.
Schol. „ Scholium.
Theor. „ Theorem.

PROPOSITIONS.

PROPOSITION I. PROBLEM.

To describe an equilateral triangle upon a given right line.



Let AB be the given line; it is required to describe an equilateral triangle upon it.

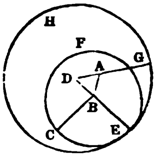
From the centre A, with the radius AB, describe the circle BCD (by Post. 3.), and from the centre B, with the radius BA, describe the circle ACE, cutting the former in C; join CA and CB (Post. 1.); then the triangle ABC is equilateral.

For the line AC is equal to AB, being radii of the same circle BCD (Def. 16.); and the line BC is equal to BA, being radii of the same circle ACE: since then each of the lines AC and BC is equal to the same line AB, they are equal to one another (Ax. 1.); therefore the three lines AB, BC, and CA are all equal; therefore the triangle ABC is equilateral; and it is described upon the given line AB. *Which was to be done.*

SCHOL. Another equilateral triangle may be obtained by drawing right lines from A and B to the point G, in which the circles cut at the other side of the line AB, which can be demonstrated in the same manner.

PROP. II. PROB.

From a given point, to draw a line equal to a given right line.



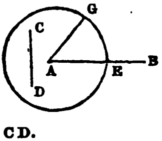
Let A be the given point, and BC the given line; it is required to draw from A a line equal to BC.

Join A and B (Post. 1.), and upon the line AB describe an equilateral triangle DAB (Prop. 1.), and from the centre B with the radius BC, describe the circle CEF (Post. 3.), and produce the line DB till it meets the circumference in E (Post. 2.); then from the centre D, with the radius DE, describe the circle EGH (Post. 3.), and produce the line DA till it meets the circumference in G: then the line AG is equal to the given line BC.

For the line DG is equal to DE, being radii of the same circle EGH (Def. 16.), and the part DA is equal to DB, being sides of an equilateral triangle (Const.), therefore the remaining part AG is equal to BE (Ax. 3.); but the line BC is also equal to BE, being radii of the same circle CEF: since then each of the lines AG and BC is equal to the same line BE, they are equal to one another(Ax. 1.); therefore the line AG is equal to the given line BC, and it is drawn from the given point A. *Which was to be done.*

PROP. III. PROB.

From the greater of two lines, to cut off a part equal to the less.



Let AB be the greater of two lines, and CD the less; it is required to cut off from AB, a part equal to CD.

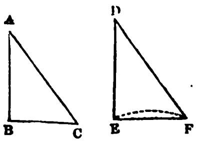
From the point A draw a line AG equal to CD (Prop. 2.), and from the centre A, with the radius AG, describe a circle (Post. 3.) cutting the line AB in E; then AE is equal to CD.

For the line AE is equal to AG, being radii of the same circle (Def. 16.); but the line CD is also equal to AG (Const.): since then each of the lines AE and CD is equal to the same line AG, they are equal to one another (Ax. 1.); therefore AE is equal to CD, and it is cut off from the greater line AB. *Which was to be done.*

COR. By a similar construction the following problem may *be solved: To* produce the less of two given lines, till it be *equal to the greater.*

PROP. IV. THEOR.

If two triangles have two sides and the contained angle in the one, respectively equal to two sides and the contained angle in the other, their bases or third sides are likewise equal, and the remaining angles of the one, are respectively equal to the remaining angles of the other, and the two triangles are equal in all respects.



Let ABC and DEF be two triangles, having the side AB equal to DE, the side AC equal to DF, and the angle A equal to the angle D; then the base BC is equal to EF, the angle B equal to the angle E, and the angle C is equal to the angle F; and the two triangles are equal in all respects.

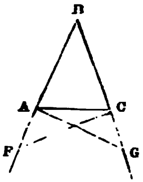
For, if the triangle ABC be so applied to the triangle DEF, that the point A may be on D, and the side AB upon DE, then the point B must coincide with E, because the sides AB and DE are equal (Hyp.); and the side AC will fall on DF, because the angles A and D are equal (Hyp.); and the point C must coincide with F, because the sides AC and DF are equal (Hyp.): and as the points B and C coincide with the points E and F, the lines BC and EF must also coincide, if not two right lines would inclose a space, which is impossible (Ax. 10.). Therefore the base BC coincides with EF, and is equal to it (Ax. 8.). And as the sides of the angle B, coincide with the sides of the angle E, the angles themselves must coincide, and are therefore equal (Ax. 8); also the angles C and F must coincide, and are therefore equal. And as the three sides of one triangle coincide with the three sides of the other, the triangles themselves must coincide, and are therefore equal in all respects (Ax. 8.). *Which was to be demonstrated.*

COR. 1. Two triangles are equal, which have an angle common to both, and the two sides which contain it in the one, equal to the two sides which contain it in the other.

COR. 2. Two triangles are equal, which have a side common to both, and another side with the contained angle in the one, equal to another side with the contained angle in the other.

PROP. V. THEOR.

The angles at the base of an isosceles triangle are equal, and if the equal sides be produced, the angles below the base are also equal.



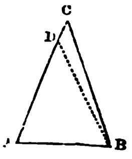
Let ABC be an isosceles triangle, having the sides AB and BC equal; then the angles BAC and BCA at the base are equal, and if the equal sides be produced, the angles FAC and GCA below the base are also equal.

Take any point F, in one of the sides produced, and cut off from the other a part BG equal to BF (Prop. 3.): join AG and FC.

Then in the triangles FCB and GAB, the sides FB and BC in the one, are equal to the sides GB and BA in the other (Hyp.), and the angle B is common to both; therefore the two triangles are equal in all respects (Prop. 4.), the base FC is equal to GA, the angle FCB to GAB, and the angle F equal to the angle G. And if from the equal sides BF and BG, there be taken the equals BA and BC, the remainders AF and CG are also equal (Ax. 3.); then in the triangles AFC and CGA, the sides AF and FC in the one, are equal to the sides CG and GA in the other, and the angle F is equal to the angle G; therefore the two triangles are equal in all respects (Prop. 4.), the angles FAC and GCA are equal, and these are the angles below the base; also the angles FCA and GAC are equal; and if from the equal angles FCB and GAB, there be taken the equals FCA and GAC, the remainders BAC and BCA are also equal (Ax. 3.), and these are the angles at the base. *Which was to be demonstrated.*

PROP. VI. THEOR.

If two angles of a triangle be equal, the sides opposite to them are also equal.



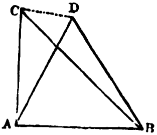
In the triangle CAB, let the angles CAB and CBA be equal; the sides CA and CB are also equal.

For if they be not equal, let one of them CA be greater than the other, and from it cut off DA equal to BC (Prop. 3.); and join BD.

Then in the triangles CAB and DAB, the sides DA and AB in the one, are equal to CB and BA in the other (Hyp.), and the angles CAB and CBA are equal (Hyp.), therefore the triangles are equal (Prop. 4.); a part equal to the whole, which is absurd; therefore neither of the sides CA and CB is greater than the other: they are therefore equal. *Which was to be demonstrated.*

Prop. VII. Theor.

If two triangles, on the same base, and on the same side of it, have the sides which terminate in one end of the base equal, those sides which terminate in the other end of the base are not equal.

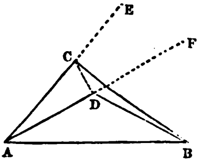


Let the triangles ACB and ADB, on the same base AB, and on the same side of it, have the sides AC and AD equal; then the sides BC and BD are not equal.

Case 1. When the vertex of each triangle is without the other.

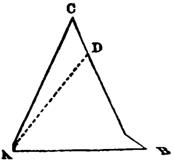
Join CD, and if it be possible, let BC be equal to BD: then in the triangle CAD, because the sides AC and AD are equal (Hyp.), the angles ACD and ADC are also equal (Prop. 5.); but the angle ACD is greater than BCD (Ax. 9.); therefore the angle ADC is greater than BCD; much more than is the angle BDC greater than BCD. But in the triangle BCD, the sides BC and BD are equal (Hyp.); therefore the angles BCD and BDC are equal (Prop. 5.); but the angle BDC has been proved to be greater than BCD; greater than, and equal to it, which is absurd; therefore the sides BC and BD are not equal.

Case 2. When the vertex D of one triangle is within the other.



Produce the sides AC and AD to E and F: then in the triangle ACD, because the sides AC and AD are equal (Hyp.), the angles DCE and CDF are also equal (Prop. 5.); but the angle DCE is greater than DCB (Ax. 9.), therefore the angle CDF is greater than DCB; much more than is the angle CDB greater than DCB. But in the triangle BCD, the sides BC and BD are equal (Hyp.), therefore the angles BDC and BCD are equal (Prop. 5.): but the angle BDC has been proved to be greater than BCD; greater than, and equal to it, which is absurd; therefore the sides BC and BD are not equal.

Case 3. When the vertex D of one triangle falls on a side of the other BC.

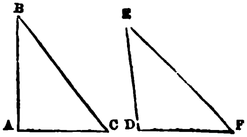


Then it is evident that the whole line BC is greater than its part BD (Ax. 9.).

Therefore in no case can the side BD be equal to BC. *Which was to be demonstrated.*

PROP. VIII. THEOR.

If two triangles have two sides of the one respectively equal to two sides of the other, and also have their bases equal, the angles opposite the equal bases are equal, and the two triangle are equal in all respects.



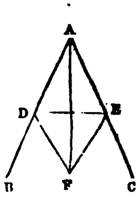
Let ABC and DEF be two triangles, having the side AB equal to DE, the side BC to EF, and the base AC equal to the base DF; then the angle B is equal to the angle E; and the two triangles are equal in all respects.

For if the triangle ABC be so applied to the triangle DEF, that the point A may be on D, and the base AC upon DF, then the point C must coincide with F, because AC and DF are equal (Hyp.); and as the base AC coincides with DF, and the sides AB and BC are equal to the sides DE and EF (Hyp.), the vertex B must fall on E (Prop. 7.), and the equal sides must coincide (Ax. 10.); therefore the angles B and E must also coincide, and are therefore equal (Ax. 8.). And as the three sides of one triangle coincide with the three sides of the other, the triangles themselves must also coincide, and are therefore equal (Ax. 8.). *Which was to be demonstrated.*

COR. Two triangles are equal, if they have a common side, and the remaining sides of the one equal to the remaining sides of the other.

PROP. IX. PROB.

To bisect a given rectilineal angle.

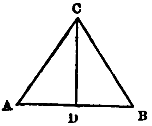


Let BAC be the given angle, it is required to bisect it.

Take any point D in the side AB, and from AC cut off a part AE equal to AD (Prop. 3.): join DE, and upon it describe an equilateral triangle DEF (Prop. 1.) at the side opposite to A; join AF; then the angle BAC or DAE is bisected by the line AF. For in the triangles DAF and EAF, the side DA is equal to EA (Const.), and AF is common to both, and the base DF is equal to EF (Const.); therefore the angle DAF is equal to EAF (Prop. 8.), and therefore the given angle BAC is bisected by the line AF. *Which was to be done.*

Prop. X. Prob.

To bisect a given right line.



Let A B be the given line, it is required to bisect it.

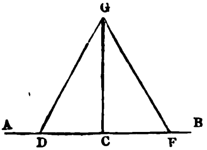
Upon A B describe an equilateral triangle A B C (Prop. I.), and bisect the angle A C B by the line C D (Prop. 9.); then A B is bisected in the point D.

For in the triangles A C D and B C D, the side A C is equal to B C (Const.), the side C D is common to both, and the angle A C D is equal to B C D (Const.); therefore the base A D is equal to B D (Prop. 4.), and therefore the given line A B is bisected in D. *Which was to be done.*

Cor. By this proposition a line may also be divided into 4 equal parts, or into 8, 16, &c., by bisecting again each part.

Prop. XI. Prob.

To draw a right line perpendicular to a given line from a given point in it.



Let A B be the given line, and C the given point; it is required to draw from C a line which shall be perpendicular to the line A B.

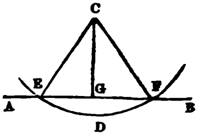
Take any point D in A C, and cut off from C B a part C F equal to C D (Prop. 3.), and upon D F describe an equilateral triangle D F G (Prop. 1.); join G C; then the line G C is perpendicular to A B.

For in the triangles C D G and C F G, the sides C D and D G in the one, are equal to the sides C F and F G in the other (Const.), and the side C G is common to both, therefore the angles D C G and F C G opposite the equal sides are equal (Prop. 8.), and these are the adjacent angles; therefore the line G C is perpendicular to A B (Def. 11.), and it is drawn from the point C. *Which was to be done.*

Cor. In like manner a perpendicular can be drawn from the extremity of a given line, by first producing the line.

Prop. XII. Prob.

To draw a right line perpendicular to a given indefinite line, from a given point not in it.



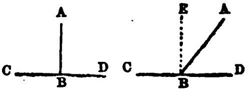
Let A B be the given line, and C the given point; it is required to draw from C, a line which shall be perpendicular to A B.

Take any point D, on the other side of A B, and from the centre C, with the radius C D, describe a circle cutting the line A B, or the line produced in the points E and F; bisect E F in the point G (Prop. 10.), and join C G; then the line C G is perpendicular to A B.

For draw the lines C E and C F; then in the triangles G E C and G F C, the side C E is equal to C F, being radii of the same circle, and E G is equal to F G (Const.), and the side C G is common to both; therefore the angles C G E and C G F opposite the equal sides are equal (Prop. 8.), and these are the adjacent angles; therefore C G is perpendicular to A B (Def. 11.), and it is drawn from the point C. *Which was to be done.*

Prop. XIII. Theor.

The angles made by one line standing upon another are together equal to two right angles.



Let the line A B stand upon C D, making with it the angles A B C and A B D; those angles are together equal to two right angles.

First. If the line A B be perpendicular to C D, then each of the angles A B C and A B D is a right angle (Def. 16.); therefore both together are two right angles.

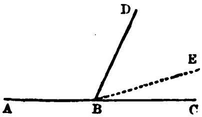
Secondly. If the line A B be not perpendicular to C D, from the point B draw B E perpendicular to C D (Prop. 11.); then it is evident that the angles A B C and A B D together are equal to the angles E B C and E B D: but E B C and E B D are two right angles (Const.), therefore A B C and A B D together are equal to two right angles. *Which was to be demonstrated.*

Cor. 1. The angles made by several lines standing upon the same right line, at the same point, are together equal to two right angles.

Cor. 2. The angles made by any number of lines meeting at a point are together equal to four right angles.

PROP. XIV. THEOR.

If two right lines meet another right line, at the same point, at opposite sides, and make angles with it equal to two right angles, the two lines form one continued right line.

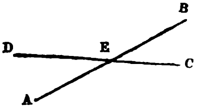


Let the lines AB and BC meet the line DB at the point B, and make with it the angles DBA and DBC equal to two right angles; then AB and BC form one right line.

For, if AB and BC do not form one right line, let any other line BE form with AB a continued right line; then, because AE is a right line and DB stands upon it, the angles DBA and DBE are equal to two right angles (Prop. 13.); but the angles DBA and DBC are also equal to two right angles (Hyp.), therefore the angles DBA and DBC are equal to DBA and DBE (Ax. 1.). Take away the angle DBA, which is common to both, and the remaining angles DBE and DBC are equal (Ax. 3.), a part equal to the whole which is absurd; therefore BE does not form a right line with AB; and in the same manner it can be proved that no other line but BC can form a right line with AB: therefore AB and BC form one continued right line. *W. W. D.*

PROP. XV. THEOR.

If two right lines cut one another, the vertical or opposite angles are equal.

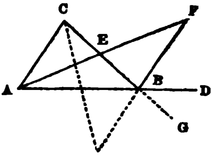


Let the lines AB and CD cut one another at E; then the angle DEB is equal to CEA, and the angle DEA is equal to BEC.

For, as the line DE stands upon BA, the angles DEB and DEA are equal to two right angles (Prop. 13.), and as the line AE stands upon DC, the angles DEA and CEA are likewise equal to two right angles; therefore the angles DEB and DEA are equal to the angles CEA and DEA (Ax. 1.). Take away the angle DEA, which is common to both, and the remaining angles DEB and CEA are equal (Ax. 3.), and these are the vertical angles. In the same manner it can be proved that the angles DEA and BEC are equal. *Which was to be demonstrated.*

PROP. XVI. THEOR.

If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles.



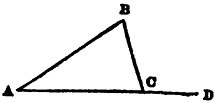
Let the side AB of the triangle ABC be produced to D; the exterior angle CBD is greater than either of the interior opposite angles CAB or ACB.

Bisect the side BC in E (Prop. 10.); draw AE and produce it till EF is equal to AE (Prop. 3.); join FB.

Then in the triangles ACE and BEF the sides AE and EC in the one, are equal to the sides FE and EB in the other (Const.), and the vertical angles AEC and BEF are equal (Prop. 15.); therefore the angles EBF and ACE are also equal (Prop. 4.); but the angle CBD is greater than CBF, therefore CBD is greater than ACE or ACB. In like manner, if the side CB be produced and AB be bisected, it can be shown that the angle ABG is greater than the interior angle CAB; but ABG is equal to CBD, being vertical angles (Prop. 15.); therefore CBD is also greater than CAB. *Which was to be demonstrated.*

PROP. XVII. THEOR.

Any two angles of a triangle are together less than two right angles.



Let ABC be a triangle; any two of its angles are together less than two right angles.

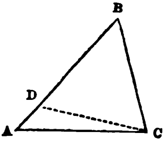
Produce the side AC to D (Cor. 2.), then the exterior angle BCD is greater than the interior angle BAC (Prop. 16.); add to each the angle BCA; and the angles BAC and BCA are less than the angles BCD and BCA; but BCD and BCA are equal to two right angles (Prop. 13.): therefore BAC and BCA are less than two right angles. In like manner, by producing the other sides, it can be shown that the angles BAC and ABC, or the angles ACB and ABC are less than two right angles. *Which was to be demonstrated.*

COR. 1. If one angle of a triangle be obtuse or right, the other two angles must be acute.

COR. 2. The angles at the base of an isosceles triangle are both acute.

PROP. XVIII. THEOR.

If one side of a triangle be greater than another, the angle opposite to the greater side, is greater than the angle opposite to the less.



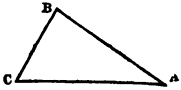
In the triangle ABC, let the side AB be greater than the side BC; then the angle BCA is greater than the angle BAC.

From the greater side AB, cut off a part BD, equal to the less BC (Prop.3.); join DC.

Because the sides BD and BC are equal (Const.), the angles BCD and BDC are eqnal (Prop.5.); but the angle BDC is greater than the interior angle BAC (Prop. 16.), therefore the angle BCD is greater than BAC; much more then is the angle BCA greater than the angle BAC. *Which was to be demonstrated.*

PROP. XIX. THEOR.

If one angle of a triangle be greater than another, the side opposite to the greater angle, is greater than the side opposite to the less.



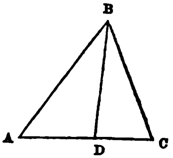
In the triangle ABC, let the angle C be greater than the angle A; then the side AB is greater than the side BC.

For the side AB is either equal to BC, or less than it, or greater than it. AB is not equal to BC, for, if it were, the angles at A and C would then be equal (Prop. 5.); but they are not (Hyp.). Again, AB is not less than BC, for, if it were, the angle C would then be less than the angle A (Prop. 18.); but it is not (Hyp.): since then AB is neither equal to, nor less than BC, it must be greater than it. *Which was to be demonstrated*

SCHOL. One side of a triangle is greater than, equal to, or less than another, according as the angle opposite to the side is greater than, equal to, or less than the angle opposite to the other.

PROP. XX. THEOR.

Any two sides of a triangle are together greater than the third side.



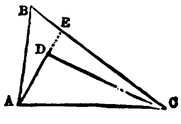
In the triangle A B C, the sides A B and B C are greater than A C, the sides B C and C A are greater than B A, and the sides C A and A B are greater than B C.

Bisect the angle A B C by the line B D (Prop. 9.).

Then the angle A D B is greater than the interior angle D B C, also the angle C D B is greater than the interior angle D B A (Prop. 16.); but the angles D B A and D B C are equal (Const.); therefore the angle A D B is greater than D B A, and the angle C D B is greater than D B C, and therefore the sides A B and B C opposite the greater angles, are greater than the sides A D and D C opposite to the less (Prop. 19.); therefore the two sides A B and B C are greater than the third side A C. In like manner, by bisecting any other angle, it can be shown that the sides containing it are greater than the third side. *W. W. D.*

PROP. XXI. THEOR.

Two right lines drawn from the ends of one side of a triangle to a point within it, are less than the two other sides of the triangle; but contain a greater angle.



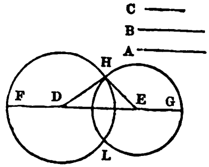
In the triangle A B C, the right lines A D and C D, drawn from A and C to the point D within the triangle, are less than the sides A B and B C; but contain an angle A D C greater than A B C.

Produce A D to E; then in the triangle A B E, the sides A B and B E are greater than the third side A E (Prop. 20.); add E C to each, then A B and B C are greater than A E and E C; but in the triangle C E D, the sides D E and E C are greater than the third side D C (Prop. 20.); add A D to each; then A E and E C are greater than A D and D C; much more then are the sides A B and B C greater than A D and D C.

Again, the exterior angle A E C is greater than the interior angle A B C (Prop. 16.), and the exterior angle A D C is greater than A E C (Prop. 16.); therefore the angle A D C is greater than A B C. *Which was to be demonstrated.*

PROP. XXII. PROB.

To construct a triangle which shall have its sides respectively equal to three given lines, of which any two are greater than the third.



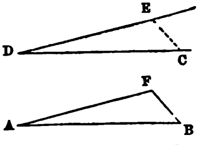
Let A, B, and C be the three given lines, of which any two are greater than the third; it is required to construct a triangle which shall have its sides respectively equal to A, B, C.

From any point D draw a line DE equal to A, and DF equal to B, and from the point E draw EG equal to C (Prop. 2.); then from the centre D with the radius DF describe a circle, and from the centre E with the radius EG describe another circle, cutting the former in H; join DH and EH; the sides of the triangle DEH are equal to the lines A, B, C. For the line DH is equal to DF, being radii of the same circle (Def. 16.); but DF is equal to B (Const.), therefore DH is equal to B (Ax. 1.). Also the line EH is equal to EG, being radii of the same circle (Def. 16.); but EG is equal to C (Const.), therefore EH is equal to C; and DE is equal to A (Const.); therefore the three sides DH, EH, and DE of the triangle DEH are equal to the given lines A, B, and C. *Which was to be done.*

SCHOL. Another triangle having its sides equal to the three given lines, may be obtained by drawing right lines from D and E to the point L, in which the circles cut at the other side of DE.

PROP. XXIII. PROB.

At a point in a given line, to make an angle equal to a given one.

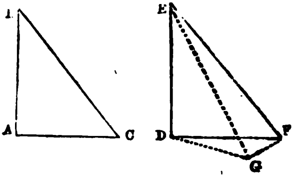


Let A be the given point, AB the given line, and D the given angle; it is required to make an angle at A equal to the angle D,

In the sides of the angle D, take any two points C and E; join CE, and construct a triangle BAF, having its sides AB, BF, and FA equal to the lines DC, CE, and ED (Prop. 22.); then the angle A is equal to the angle D (Prop. 8). *W. W. D.*

PROP. XXIV. THEOR.

If two triangles have two sides of the one, respectively equal to two sides of the other, and if the contained angle in the one be greater than the contained angle in the other; the base opposite to the greater angle is greater than the base opposite to the less.

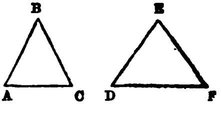


In the triangles A B C and D E F, let the side A B be equal to D E, the side B C to E F: but the angle D E F greater than the angle A B C; then the base D F is greater than A C.

At the point E with the less side D E, make an angle D E G equal to the angle A B C (Prop. 23.), and make E G equal to B C or E F (Prop. 3.); join G F and G D. Then in the triangles A B C and D E G, the sides A B and B C in the one, are equal to the sides D E and E G in the other (Const.), and the angle B is equal to the angle D E G (Const.); therefore the base D G is equal to A C (Prop. 4.). And as the sides E F and E G are equal (Const.), the angles E G F and E F G are equal (Prop. 5.); but the angle E F G is greater than D F G, therefore the angle E G F is greater than D F G; much more then is the angle D G F greater than D F G: therefore in the triangle D F G, the side D F is greater than D G (Prop. 19.); but D G is equal to A C, therefore D F is greater than A C. *W. W. D.*

PROP. XXV. THEOR.

If two triangles have two sides of the one respectively equal to two sides of the other, and if the base of one be greater than the base of the other; the angle opposite to the greater base is greater than the angle opposite to the less.

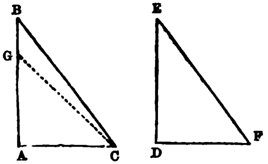


In the triangles A B C and D E F, let the side A B be equal to D E, and B C to E F; but the base D F greater than A C: the angle E is greater than the angle B.

For the angle E is either less than, equal to, or greater than the angle B. It is not less than it, because the base D F would then be less than A C (Prop. 24.), but it is not (Hyp.). It is not equal to it, because the bases D F and A C would then be equal (Prop. 4.), but they are not (Hyp.): since then the angle E is neither less than, nor equal to the angle B, it must be greater than it. *W. W. D.*

Prop. XXVI. Theor.

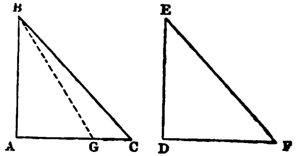
If two triangles have two angles and a side in the one, respectively equal to two angles and a side similarly situated in the other, the remaining sides and angle in the one, are respectively equal to the remaining sides and angle in the other, and the two triangles are also equal.



In the triangles A B C and D E F, let the angle B A C be equal to E D F, the angle A C B to D F E, and a side in one triangle, equal to a side similarly situated in the other; the remaining sides and angles are respectively equal.

Case 1. When the equal sides are A C and D F adjacent to the equal angles, then the side A B is equal to D E, and the triangles are also equal.

For, if A B and D E be not equal, let one of them A B be greater than the other, and from it cut off A G equal to D E, and join C G. Then the sides A C and A G are equal to the sides D F and D E, and the angles at A and D are equal (Hyp.); therefore the angle A C G is equal to D F E (Prop. 4.): but the angle A C B is also equal to D F E (Hyp.); therefore A C G is equal to A C B, a part equal to the whole, which is absurd; therefore A B and D E are not unequal, therefore they are equal; and also A C is equal to D F, and the angles at A and D are equal (Hyp.); therefore B C is equal to E F, the angle B to the angle E, and the triangles A B C and D E F are equal in all respects (Prop. 4.).



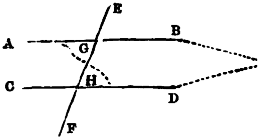
Case 2. When the equal sides are A B and D E opposite to the equal angles, then A C is equal to D F.

For, if A C and D F be not equal, let one of them A C be greater than the other, and from it cut off A G equal to D F, and join B G. Then the sides A G and A B are equal to the sides D F and D E, and the angles at A and D are equal (Hyp.); therefore the angle A G B is equal to D F E (Prop. 4.); but the angle A C B is also equal to D F E (Hyp.), therefore A G B is equal to A C B, an exterior angle equal to its interior opposite angle, which is impossible (Prop. 16.); therefore A C and D F are not unequal, therefore they are equal; and also A B is equal to D E, and the angles at A and D are equal (Hyp.); therefore B C is equal to E F, the angle B to the angle E, and the triangles A B C and D E F are equal in all respects (Prop. 4.). W. W. D.

Prop. XXVII. Theor.

If a right line fall upon two other right lines, and make alternate angles equal to one another, the two lines are par

Let the line E F fall upon the two lines A B and C D, and make the alternate angles A G F and E H D equal; then A B and C D are parallel.

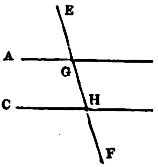


For, if A B and C D be not parallel, they would meet if produced. Suppose them to meet in L; then G H L is a tria and the exterior angle A G H is greater than the interior a E H D (Prop. 16.); but these angles are also equal (H which is absurd; therefore A B and C D do not meet if prod and are therefore parallel (Def. 30.). *Which was to be de strated.*

Prop. XXVIII. Theor.

If a right line fall upon two other right lines, and make an terior angle equal to the interior and opposite angle at the side of the line: or make the two interior angles at the same equal to two right angles; the two lines are parallel.

Let the line E F fall upon the two lines A B and C D, and make the exterior angle E G B equal to the interior opposite angle E H D at the same side of E F: or make the two interior angles B G F and E H D at the same side equal to two right angles; then A B and C D are parallel.

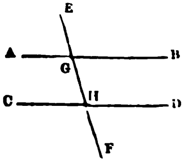


For the angle A G F is equal to E G B (Prop. 15.), and the angle E G B is equal to E H D (Hyp.), therefore the angles A G F and E H D are e (Ax. 1.); but they are alternate angles, therefore A B an are parallel (Prop. 27.).

Again: The angles B G F and A G F are equal to two angles (Prop. 13.); but B G F and E H D are also equal to right angles (Hyp.), therefore the angles B G F and A G equal to the angles B G F and E H D; take away the angle common to both, and the remaining angles A G F and are equal (Ax. 3.); but they are alternate angles, ther A B and C D are parallel. *W. W. D.*

PROP. XXIX. THEOR.

If a right line fall upon two parallel right lines, it makes the alternate angles equal to one another, and the exterior angle equal to the interior opposite angle at the same side; and also the two interior angles at the same side, equal to two right angles.



Let the line E F fall upon two parallel lines A B and C D; then the alternate angles A G F and E H D are equal: and the exterior angle E G B equal to the interior angle E H D at the same side of E F: and also the two interior angles B G F and E H D at the same side equal to two right angles.

Part 1. The alternate angles A G F and E H D are equal.

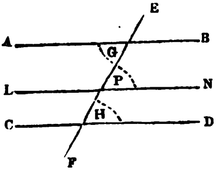
For, if the angles A G F and E H D be not equal, one of them must be greater than the other; suppose A G F to be greater, and add the angle B G F to each, then the angles A G F and B G F together are greater than the angles E H D and B G F; but A G F and B G F are equal to two right angles (Prop. 13.); therefore E H D and B G F are less than two right angles, and therefore the lines A B and C D, if produced, would meet at the side B D (Ax. 12.): but they are parallel, and therefore cannot meet (Def. 30.), which is absurd; therefore neither of the angles A G F and E H D is greater than the other, therefore they are equal.

Part 2. The exterior angle E G B is equal to the interior opposite angle E H D. For the angles A G F and E G B are equal, being vertical opposite angles (Prop. 15.), and the angles A G F and E H D are equal, being alternate angles (Part 1.); since then each of the angles E G B and E H D is equal to the same angle A G F; they are equal to one another (Ax. 1.); therefore the angles E G B and E H D are equal.

Part 3. The two interior angles B G F and E H D are equal to two right angles. For the angles B G F and A G F are equal to two right angles (Prop. 13.); but A G F is equal to E H D, being alternate angles (Part 1.); therefore the angles B G F and E H D are equal to two right angles. *Which was to be demonstrated.*

PROP. XXX. THEOR.

If two right lines be parallel to the same right line, they are parallel to one another.



Let the lines A B and C D be parallel to the same line L N; then A B and C D are parallel to one another.

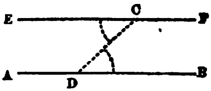
For the angles A G F and E P N are equal, being alternate angles (Prop. 29.); but the angle E P N is equal to the interior opposite angle E H D (Prop. 29.); therefore the angles A G F and E H D are equal (Ax. 1.), and they are alternate angles; therefore the lines A B and C D are parallel (Prop. 27.). *Which was to be demonstrated.*

SCHOL. This proposition may be demonstrated without the aid of the 27th, thus: — Each of the angles E G B and E H D is equal to the same angle E P N (Prop. 29.); therefore they are equal to one another (Ax. 1.); and therefore the lines A B and C D are parallel (Prop. 28.).

COR. Two lines parallel to the same line, cannot pass through the same point.

PROP. XXXI. PROB.

To draw a right line parallel to a given right line, through a given point.

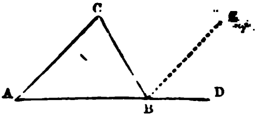


Let A B be the given line, and C the given point; it is required to draw through C, a line parallel to A B.

In the line A B take any point D; join C D, and with it at the point C make an angle D C E equal to C D B (Prop. 23.), but at the opposite side of C D; then the line E F is parallel to A B. For, as the line C D falls upon the two lines A B and E F, and makes the alternate angles E C D and B D C equal to one another (Con.); therefore the two lines A B and E F are parallel (Prop. 27.). *Which was to be done.*

PROP. XXXII. THEOR.

If any side of a triangle be produced, the exterior angle is equal to the two interior opposite angles: and the three interior angles of every triangle, are equal to two right angles.



In the triangle ABC, let the side AB be produced to D; then the exterior angle CBD is equal to the two interior opposite angles CAB, and ACB: and the three interior angles CAB, ACB, and CBA together, are equal to two right angles.

From the point B draw BE parallel to AC (Prop. 31.): then, because the line BC falls upon the parallels AC and BC, the alternate angles ACB and EBC are equal (Prop. 29.); also, because AD falls upon the same parallels, the angle EBD is equal to the interior angle CAB (Prop. 29.); therefore the whole exterior angle CBD is equal to the interior opposite angles CAB and ACB.

Again: Because the angles CBA and CBD are equal to two right angles (Prop. 13.), but CBD was shown to be equal to the two interior angles CAB and ACB; therefore the three interior angles, CBA, CAB, and ACB, are equal to two right angles. *W. W. D.*

COR. 1. If one angle of a triangle be equal to the other two angles, it is a right angle.

COR. 2. A perpendicular may be drawn from the end of a line, by describing an equilateral triangle upon it as a base, and producing one of the sides till the part produced be equal to the base, and joining the end of the produced line to the end of the base.

COR. 3. All the interior angles of any rectilineal figure, together with four right angles, are equal to twice as many right angles as the figure has sides.

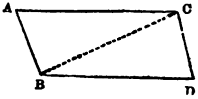
COR. 4. All the exterior angles of any rectilineal figure are together equal to four right angles.

COR. 5. Each angle of an equilateral triangle is equal to a third of two right angles, or two-thirds of one right angle.

COR. 6. A right angle may be trisected, by describing an equilateral triangle on one of its sides, and bisecting that angle of the triangle which is at the vertex of the right angle.

PROP. XXXIII. THEOR.

The right lines which join the adjacent extremities of two equal and parallel lines, are themselves equal and parallel.

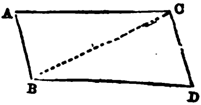


Let A B and C D be two equal and parallel lines; the right lines A C and B D joining the adjacent extremities A to C, and B to D, are also equal and parallel.

Draw the diagonal B C. And because the line B C falls on the parallels A B and C D, the alternate angles B C D and A B C are equal (Prop. 29.); then in the triangles A B C and B C D, the side A B is equal to C D, and B C is common to both, and the angle A B C is equal to B C D; therefore the side A C is equal to B D (Prop. 4.), and the angle A C B is equal to D B C; but they are alternate angles, therefore the lines A C and B D are parallel (Prop. 27.). *W. W. D.*

PROP. XXXIV. THEOR.

The opposite sides and angles of a parallelogram are equal to one another, and the diagonal divides it into two equal parts.

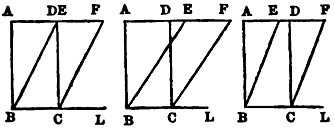


Let A B C D be a parallelogram, and B C the diagonal; the side A B is equal to C D, and A C to B D; the angles at A and D are equal, also the angles at B and C are equal; and the diagonal B C divides the parallelogram into two equal parts.

Because the line B C falls on the parallels A B and C D, the alternate angles A B C and B C D are equal (Prop. 29.); also because B C falls on the parallels A C and B D, the alternate angles A C B and D B C are equal (Prop. 29.); then in the triangles A B C and B C D, the angles A B C and A C B in the one, are equal to the angles B C D and D B C in the other, and the side B C is common to both; therefore the two triangles are equal in all respects (Prop. 26.), the side A B is equal to C D, and A C to B D, and the angles at A and D are equal; also the two angles A C B and B C D are respectively equal to the two angles D B C and A B C (Prop. 26.); therefore the whole angle A C D is equal to the whole angle A B D. And as the triangles A B C and B C D are equal, the parallelogram is divided into two equal parts by the diagonal B C. *W. W. D.*

PROP. XXXV. THEOR.

Parallelograms upon the same base and between the same parallels, are equal.



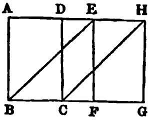
Let the parallelograms ABCD and EBCF, be on the same base BC, and between the same parallels AF and BC; they are equal to one another.

For, produce the base BC to L; and because the line BL falls on the parallels AB and DC, the angles ABC and DCL are equal (Prop. 29.); also, because BL falls on the parallels EB and FC, the angles EBC and FCL are equal (Prop. 29.), therefore the angles ABE and DCF are also equal (Ax. 3.); and the lines which contain them are equal, AB to DC and EB to FC (Prop. 34.); therefore the triangles ABE and DCF are equal (Prop. 4.): take away each of these equal triangles respectively from the quadrilateral figure ABCF, and the remainders are the parallelograms ABCD and EBCF, which are equal (Ax. 3.). *W. W. D.*

SCHOL. On this proposition is founded the rule in Mensuration for finding the area of any parellelogram; namely, multiply the length of the base by the perpendicular breadth, and the product will be the area.

PROP. XXXVI. THEOR.

Parallelograms upon equal bases and between the same parallels are equal.

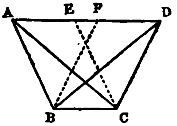


Let the parallelograms ABCD and EFGH be upon equal bases BC and FG, and between the same parallels AH and BG: they are equal.

Join BE and CH; then the line EH is equal to FG (Prop. 34.), and BC is also equal to FG (Hyp.), therefore EH and BC are equal to one another: but they are also parallel (Hyp.); therefore BE and CH are also equal and parallel (Prop. 33.); therefore BCHE is a parallelogram (Def. 35.); and it is equal to each of the parallelograms ABCD and EFGH, being on the same base and between the same parallels (Prop. 35.); therefore the parallelograms ABCD and EFGH are equal (Ax. 1.). *W. W. D.*

PROP. XXXVII. THEOR.

Triangles upon the same base and between the same parallels are equal.

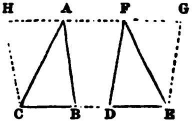


Let the triangles B A C and B D C be on the same base B C, and between the same parallels B C and A D: they are equal.

Through the point C draw the line C E parallel to B A (Prop. 31.), and through B draw B F parallel to C D: then the parallelograms A B C E and F B C D are equal, being on the same base B C, and between the same parallels (Prop. 35.); but the triangles B A C and B D C are halves of them (Prop. 34.), and are therefore equal. *Which was to be demonstrated.*

PROP. XXXVIII. THEOR.

Triangles upon equal bases and between the same parallels are equal.



Let the triangles B A C and D F E be upon equal bases B C and D E, and between the same parallels A G and B E; the triangles B A C and D F E are equal.

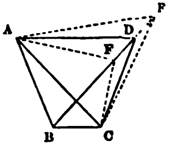
Through the point C draw the line C H parallel to A B (Prop. 31.), and through E draw E G parallel to D F; then the parallelograms A B C H and F D E G are equal, being on equal bases and between the same parallels (Prop. 36.); but the triangles B A C and D F E are halves of them (Prop. 34.), and are therefore equal. *Which was to be demonstrated.*

COR. 1. A line drawn from the vertex of a triangle bisecting the base, bisects the triangle.

COR. 2. A triangle may be divided into any number of equal parts, by dividing the base into the given number of parts, and drawing right lines to the vertex of the triangle.

PROP. XXXIX. THEOR.

Equal triangles upon the same base, and upon the same side of it, are between the same parallels.

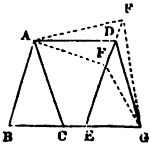


Let the equal triangles BAC and BDC be upon the same base BC, and on the same side of it: they are between the same parallels.

Join the points A and D; then, if AD be not parallel to BC, draw through A a line AF parallel to BC (Prop. 31.), cutting BD or BD produced in F: join CF; then, because the triangles BAC and BFC are upon the same base BC, and between the same parallels BC and AF, they are equal (Prop. 37.); but the triangle BAC is equal to BDC (Hyp.); therefore the triangles BFC and BDC are equal, a part equal to the whole, which is absurd: therefore AF is not parallel to BC; in the same manner it can be shown that no other line but AD is parallel to BC; therefore AD and BC are parallel. *W. W. D.*

PROP. XL. THEOR.

Equal triangles upon equal bases, in the same right line, and on the same side, are between the same parallels.

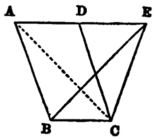


Let the equal triangles BAC and EDG be upon equal bases BC and EG, in the same line BG, and on the same side of it: they are between the same parallels.

Join the points A and D; then if AD be not parallel to BG, draw through A a line AF parallel to BG (Prop. 31.), cutting ED or ED produced in F: join GF; then, because the triangles BAC and EFG are upon equal bases BC and EG, and between the same parallels BG and AF, they are equal, (Prop. 38.); but the triangle BAC is equal to EDG (Hyp.); therefore the triangles EFG and EDG are equal, a part equal to the whole, which is absurd; therefore AF is not parallel to BG; in the same manner it can be shown that no other line but AD is parallel to BC; therefore AD and BG are parallel. *W. W. D.*

PROP. XLI. THEOR.

A parallelogram is double of a triangle upon the same base and between the same parallels.



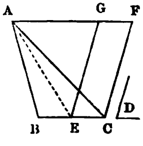
Let the parallelogram A B C D and the triangle B E C be upon the same base B C, and between the same parallels A E and B C: the parallelogram is double of the triangle.

Join A C: then, because the triangles B A C and B E C are upon the same base B C, and between the same parallels A E and B C, they are equal (Prop. 37.); but the parallelogram A B C D is double of the triangle B A C (Prop. 34.); therefore the parallelogram A B C D is also double of the triangle B E C. *Which was to be demonstrated.*

SCHOL. Hence the rule in Mensuration for finding the area of a triangle: multiply the length of the base by the perpendicular breadth of the triangle, and half the product will be the area.

PROP. XLII. PROB.

To describe a parallelogram equal to a given triangle, and having an angle equal to a given one.

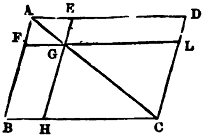


Let B A C be the given triangle, and D the given angle. It is required to describe a parallelogram equal to the triangle B A C, and having an angle equal to D.

Draw through A a line A F parallel to B C: bisect the base B C in E, and with the line E C make an angle C E G equal to the given angle D (Prop. 23.): from C draw C F parallel to E G, and join A E: then, because G F is parallel to E C, and C F parallel to E G (Const.), E C F G is a parallelogram (Def. 35.): and because the triangles B A E and E A C are upon equal bases B E and E C, and between the same parallels, they are equal (Prop. 38.); therefore the whole triangle B A C is double of one of them E A C: but the parallelogram E C F G is also double of the triangle E A C (Prop. 41.); therefore the parallelogram E C F G is equal to the triangle B A C (Ax. 6.); and it has an angle C E G equal to the given angle D (Const.) *W. W. D.*

Prop. XLIII. Theor.

In a parallelogram the complements of the parallelograms about the diagonal are equal.

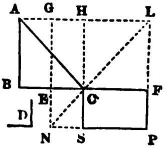


Let ABCD be a parallelogram, and AC the diagonal; also let FE and HL be the parallelograms about the diagonal; then the complements BG and GD which complete the figure are equal.

Because each parallelogram is divided by its diagonal into two equal parts (Prop. 34.), the triangle CAB is equal to CAD, the triangle GAF to GAE, and the triangle CGH to CGL; therefore the two triangles GAE and CGL are equal to the two triangles GAF and CGH; take away these equals from the equal triangles CAD and CAB, and the remainders are the complements BG and GD, which are equal (Ax. 3.). *W. W. D.*

Prop. XLIV. Prob.

To a given right line to apply a parallelogram, equal to a given triangle, and having an angle equal to a given one.



Let ABC be the given triangle, CF the given line, and D the given angle; it is required to apply a parallelogram to the line CF, equal to the triangle ABC, and having an angle equal to the angle D.

Place one side BC of the triangle in the same line with CF so that they form a continued right line; bisect BC in E, and make the parallelogram CEGH equal to the triangle ABC, and having an angle ECH equal to the angle D (Prop. 42.); through F draw FL parallel to CH or EG, and produce GH to L: draw LC to meet GE produced to N; through N draw NP parallel to EF or GL; produce LF and HC to P and S.

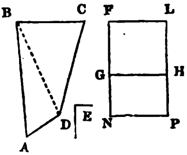
Then in the parallelogram GP, the complements GC and CP are equal (Prop. 43.); but GC is equal to the triangle ABC (Const.), therefore CP is equal to the triangle ABC.

And because the angle ECH is equal to the angle FCS (Prop. 15.), and also to the angle D, therefore the angle FCS is equal to the angle D (Ax. 1.).

Therefore the parallelogram CP is applied to the line CF, is equal to the triangle ABC, and has an angle FCS equal to the given angle D. *W. W. D.*

PROP. XLV. PROB.

To describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given one.



Let ABCD be the given rectilineal figure, and E the given angle; it is required to describe a parallelogram equal to the figure ABCD, and having an angle equal to E.

Divide the given figure into triangles by joining the opposite angles B and D, and describe a parallelogram FGHL equal to the triangle BCD, and having an angle LFG equal to the given angle E (Prop. 42.): to the side GH apply a parallelogram GNPH equal to the triangle BAD, and having an angle HGN equal to LFG (Prop. 44.): then FNPL is a parallelogram equal to the given figure ABCD, and has an angle LFN equal to the given angle E.

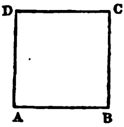
Because the lines FL and GH are parallel, the angles LFG and HGF are equal to two right angles (Prop. 29.); but the angle HGN is equal to LFG (Const.), therefore HGN and HGF are equal to two right angles, and therefore the lines FG and GN form one right line (Prop. 14.). And because the lines LH and FN are parallel, the angles GHL and FGH are equal to two right angles (Prop. 29.); but the angle FGH is equal to GHP, being alternate angles (Prop. 29.), therefore GHL and GHP are equal to two right angles, and therefore the lines LH and HP form one right line (Prop. 14.). And because the lines FL and NP are parallel to the same line GH, they are parallel to one another (Prop. 30.); therefore LFNP is a parallelogram (Def. 35.), and it is equal to the given rectilineal figure ABCD (Const.), and it has the angle LFN equal to the given angle E. *Which was to be done.*

COR. 1. A parallelogram can be described equal to the sum of two or more rectilineal figures by dividing them into triangles.

COR. 2. A parallelogram can also be described equal to the difference between two rectilineal figures.

Prop. XLVI. Prob.

On a given right line to describe a square.



Let AB be the given line, it is required to describe a square upon it.

From the point A draw AD perpendicular to AB (Prop. 11.), and equal to it (Prop. 3.); through B draw BC parallel to AD, and through D draw DC parallel to AB (Prop. 31.); then ABCD is the required square.

Because ABCD is a parallelogram (Const.), the sides BC and CD are equal to their opposite sides AD and AB (Prop. 34.); but AD and AB are equal to one another (Const.), therefore the four sides AB, AD, DC, and CB are equal. And because the angles BAD and ADC are equal to two right angles (Prop. 29.), and BAD is a right angle (Const.), therefore ADC is a right angle; but the opposite angles of a parallelogram are equal to one another (Prop. 34.); therefore the opposite angles ABC and BCD are right angles, and therefore ABCD is a square (Def. 31.). *Which was to be done.*

Cor. 1. If two right lines be equal, their squares are equal.

Cor. 2. If two squares be equal, their sides are equal.

Cor. 3. If one angle of a parallelogram be a right angle, all its angles are right angles.

Cor. 4. The diagonals of a square are equal, and bisect one another at right angles.

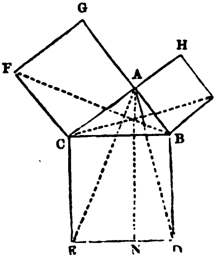
Cor. 5. If the difference between the side of a square and its diagonal be given, the square can be constructed.

Cor. 6. The diagonal of a square divides it into two isosceles triangles.

Cor. 7. If the diagonal of a square be given, the square can be constructed.

PROP. XLVII. THEOR.

In a right-angled triangle, the square described upon the si opposite to the right angle is equal to the squares described up the sides which contain the right angle.



Let ABC be a right-angled triangle, having the right angle BAC; the square described on BC is equal to the squares described upon the sides BA and AC.

On the sides BC, BA, and AC describe the squares BE, BH, and CG (Prop. 46.); through A draw AN parallel to BD or CE, and join AE and BF.

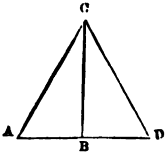
Then, because the angles BAC and GAC are two right angles, the lines BA and AG form one right line (Prop. 14.); and because t angles ACF and BCE are equal (Ax. 11.), if BCA be added each, the angles BCF and ACE are equal; and the sides and CF are equal to the sides EC and CA (Def. 31.); therefo the triangles BCF and ECA are equal (Prop. 4.): but t square CG is double of the triangle BCF, being on the same ba CF and between the same parallels (Prop. 41.), and the parallel gram CN is double of the triangle ECA, being on the same b CE and between the same parallels (Prop. 41); therefore t square CG is equal to the parallelogram CN, being doubles of t equal triangles (Ax. 6.). In the same manner it can be sho that the square BH is equal to the parallelogram BN, by joini AD and CL; therefore the whole square BDEC is equal to t two squares CG and BH. *Which was to be demonstrated.*

SCHOL. 1. In a right-angled triangle, the side opposite to t right angle is called the hypotenuse, and the other sides called the base and perpendicular.

SCHOL. 2. On this proposition is founded the rule in Mens ration for finding any side of a right-angled triangle when t other two sides are given. To find the hypotenuse: — Add t square of the base to the square of the perpendicular, a extract the square root of the sum. To find the base: — Ta the square of the perpendicular from the square of the hy tenuse, and extract the square root of the difference. To fi the perpendicular: — Take the square of the base from square of the hypotenuse, and extract the square root of *difference.*

PROP. XLVIII. THEOR.

If the square described upon one side of a triangle be equal to the squares described upon the other two sides, the angle opposite to that side is a right angle.



In the triangle ABC, let the square of AC be equal to the squares of AB and BC; then the angle ABC is a right angle.

From B draw BD perpendicular to BC (Prop. 11.), and equal to AB (Prop. 3.), and join CD.

Because the angle DBC is a right angle, the square of CD is equal to the squares of BC and BD (Prop. 47.); but BD is equal to AB, therefore the square of CD is equal to the squares of BC and AB: but the square of AC is also equal to the squares of BC and AB (Const.); therefore the squares of AC and CD are equal (Ax. 1.), and therefore the side AC is equal to CD: and because AB is also equal to BD, and BC common, the triangles BAC and BDC are equal (Prop. 8.); therefore the angle ABC is equal to the angle DBC: but DBC is a right angle (Const.), therefore ABC is also a right angle. *Which was to be demonstrated.*

EXERCISES.

1. Draw a perpendicular from the end of a line, without producing the line.
2. Show that every equilateral triangle is also equiangular.
3. If a line bisect the angle opposite to the base of an isosceles triangle, it bisects the base also.
4. Two perpendiculars cannot be drawn from the same point to the same right line.
5. The angles made by any number of lines meeting in a point are equal to four right angles.
6. All the interior angles of any rectilineal figure, are equal to twice as many right angles as the figure has sides, less four right angles.
7. The exterior angles of a rectilineal figure are equal to four right angles.
8. The parallelograms about the diagonal of a square are squares.
9. To bisect a right angle.
10. To bisect a right line.
11. The diagonals of a parallelogram divide it into four equal parts.
12. In a square the diagonals cross one another at right angles.
13. To bisect a triangle.
14. To bisect a square.
15. Given the base and perpendicular of a right-angled triangle, to find the hypotenuse.
16. To find a square equal to the sum of three squares.
17. To describe a square, when the diagonal is given.
18. If a triangle and a parallelogram be between the same parallels, and the base of the triangle be double the base of the parallelogram, the triangle and parallelogram are equal.
19. If two right lines be equal, their squares are equal.
20. Every parallelogram having one angle right, has all its angles right angles.

THE END.

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